NEGATION, DENIAL AND FALSITY : LOGIC'S NEGATIVE TRIO

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Negation lies at the heart of numerous debates about logic. Think, for instance, about the question whether the law of excluded middle holds, so important in turn to debates around realism and anti-realism; or think about discussions of contradictions and what follows from them. Negation is a logical connective, an expression which occurs in various languages. It bears some kind of relation to falsity, which is a truth-value, and to the speech-act of denial. Perhaps getting clear about the nature of these relations will shed light on negation, and thereby on the debates in which it figures. The business of this paper is the more modest first one, getting clear about the relations between the items in logic's negative trio – negation, denial and falsity. This is undertaken in the hope that this will aid progress in other areas, but that is business for elsewhere.[1]

§1 below lays out what is effectively the orthodox account of the relationship between negation and denial, owing to Frege and Geach. Drawing on Dummett's work on truth, §2 develops a challenge to this account, that it does not permit an adequate understanding of falsity. §3 explores the approach to denial common amongst dialetheists, and finds this similarly unable to account for falsity, as well as offering an unacceptably purely formal understanding of negation. §4 draws on what has gone before and presents an alternative understanding of the relationship between negation, denial and falsity, in the hope that this will better facilitate progress with respect to those philosophical disputes in which these concepts feature.

1. THE FREGE-GEACH ORTHODOXY

If there is an orthodoxy concerning denial it is that the denial of P is to be identified with the assertion of $\sim P$.[2] Denial is simply a special case of assertion and is not to be understood as consisting in the articulation of thoughts with a distinctive force. On this view there is a straightforward account to be had of the relationship between denial and negation, and it is one on which negation explains denial rather than *vice-versa*. The position was given initial motivation by Frege and subsequently championed by Geach [6] [7]. It is with them that any critical reassessment of the orthodoxy must begin.

Frege draws our attention to the fact that thoughts are expressed using sentences containing negating expressions, that the sense of a negation may be part of a content. We cannot understand negation thus described to be the opposite of assertion, since assertion is a force attaching to contents rather than a constituent of content itself. If we hold that if there *is* a force of denial, negation must be an operator expressing it. It follows quickly that there cannot be any such force. Were it otherwise there would be a violation of, what Geach terms, 'the Frege point',

> A thought may have just the same content whether you assent to its truth or not;
> a proposition may occur in discourse now asserted, now unasserted, and yet be
> recognisably the same proposition. [7, 449]

To see what is at issue here consider first an example not involving negation directly. The conditional proposition $P \rightarrow Q$ contains P as a constituent. But in asserting $P \rightarrow Q$ a language-user

[1]In [] I've suggested that a rejectivist understanding of negation, of the sort developed here, might underwrite an unorthodox paraconsistency.

[2]p. 3 Berto 2014 and references, good on this.

does not assert the proposition P. On the contrary, she may very well be committed to *denying P*. Imagine somebody engaged in hypothetical reasoning in dialogue with an opponent who believes that P. She asserts the conditional, perhaps in order to move towards arguing by *modus tollens* that $\sim P$. In order for her reasoning to have any suasive force it is essential that the content of P is constant whether or not the proposition is asserted. Such stability of content between assertoric and non-assertoric contexts is a pre-requisite for rational communication.

Now, if negation were a force operator, the stability requirement would be violated. The sense of P is distinct from that of $\ulcorner \sim P \urcorner$; obviously so because the two always differ in truth-value, which could not be the case if there were no difference in sense. If follows that the negation in $\ulcorner \sim P \urcorner$ is not signalling the attachment of a non-assertoric force to the proposition P, but rather serves to form a sentence expressing a distinct, complex, proposition. Moreover, that negation is part of content as distinct from force is further indicated by the fact that negations[3] retain constant content over assertoric and non-assertoric contexts. As Geach puts the matter,[4]

> Negation often gets paired off with assertion as its polar opposite; this is another mistake over the Frege point- one exposed by Frege himself in his paper, *Negation*. Just as I can put forward a proposition 'S' without asserting 'S' as true, so I can put forward the negation of "S" without rejecting 'S' as false- for example, when this negation occurs as part of a longer proposition, in a context, say, of the form 'P and Q, or else R and not S.' Thus logic in any case demands the use of a negation sign which is not polarly opposed to the assertion sign and does not express rejection of what is negated; and when a proposition is rejected, we may equally well conceive this as asserting the negation of a proposition. [7, 454]

It is surely correct that negation is not a force operator. In particular, then, if assertion has a 'polar opposite' it is not negation. Yet it does not follow that assertion has no such opposite, merely that if it does it is not negation. Why should Frege and Geach's considerations concerning negation cause anyone to assent to the stronger claim, not only that negation does not express a distinctive force of denial, but that there is *no* force of denial?

The reasoning of both authors at this point has a contemporary feel to it, putting considerations of theoretical economy in the foreground. Once a language contains a means of expressing negation, there is no need to postulate a distinctive force of denial since a proposition can be denied simply by asserting the negation of that proposition. Denial, on this view, is a special case of assertion and therefore a phenomenon at the level of content rather than force. To hold otherwise, thinks Geach, would involve the logician in 'futile complications'. For,

> All we need in logic for assertion and negation is two signs-the assertion sign, and a negation which does not convey rejection (as in 'if not Q . . .') [W]hatever is more than these, as Frege says, cometh of evil.[7, 454-455]

As this indicates, Frege also thinks that economy favours an understanding of denial in terms of negation [6, 361].

What is being understood here by, in Geach's phrase, the *needs* of logic? Given a classical or intuitionistic background (but not all paraconsistent or paracomplete ones), there would seem to be no loss of deductive capacity involved in the absence of a force of denial from our linguistic re-sources. Whenever a participant in an argument would want to deny a proposition, where denial is understood as a distinctive speech act, she can simply assert the negation of the proposition. Indeed the equivalence of bivalent and univalent proof systems for classical and intuitionist logic assure us that no inferential loss is incurred by such frugality with forces. However, logicians and philoso-phers have ambitions beyond the mere collection of valid arguments. We want to explicate why it is that valid arguments are important, how validity relates to the meaning of logical vocabulary, what

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[3]That is, propositions containing negation as a constituent.

[4]I have capitalised the propositional letters in the quotation.

the relationship is between central logical concepts, and what a grasp of these concepts consists in. And with respect to this more ambitious understanding of the logical task, the Frege-Geach orthodoxy falls short. We are told that denial is an assertion of a negated proposition, but this offers us no understanding of what negation itself is. Unless I know what is involved in asserting $\sim P$, for any given P, I am no closer to an understanding of denial. Nor are we offered any insight into the concept of *falsity*, which one might have expected to be intimately involved in an account of denial (compare here assertion and truth). This last point is vital to the sequel.

Before turning our attention to falsity, it ought to be acknowledged that a proponent of the Frege-Geach position, if she is a logical inferentialist,[5] can offer an account of negation and so expand her explication of denial. An understanding of negation, on this account, consists in at least an implicit grasp of the rules of use as represented in a natural deduction system. Now there are notorious problems supplying a satisfactory pair of introduction and elimination rules if the desire is to represent *classical* negation. Since Prior's discussion of the 'tonk' connective we have known that not all pairs of rules can be accepted as encoding the sense of a meaningful connective [13]. In response to the challenge of delineating the acceptable pairs of rules, many logicians have insisted on a harmony constraint for acceptable pairs of rules [][5]. The debate is familiar and covered elsewhere.[6] Suffice it to say that there is ongoing controversy concerning whether harmonious rules can be supplied for classical, as distinguished from intuitionistic, negation.[7] Supposing that either acceptable rules are forthcoming for classical negation, or else that our proponent of the Frege-Geach position is content to view negation intuitionistically, however, denial can be described as the assertion of a negation, with negation in turn being given a use-theoretic account. To deny a proposition just is to assert the negation of that proposition, and to be able to assert negations a language-user needs a grasp of the rules governing the connective. Thus to be capable of denial is to be capable of using negation.

If this were correct would it constitute an adequate account of denial? I think that it wouldn't and this is because a central aspect of both negation and denial is missing, namely the relation of each to falsity. The next section will expand on this charge; the ultimate aim of this paper is to supply an alternative account of the relationship between negation and denial, one which does not lack something to say about falsity.

2. THE PUZZLE OF FALSITY

In order to think well about falsity it is worth revisiting a key moment in analytic discussion of truth (truth has received a good deal more attention than falsity, which seems at least *prima facie* odd given our epistemic goals, not only to seek truth but to avoid falsehood.) In his seminal essay *Truth*, Dummett writes about the concept of *winning* and its relationship to the practice of playing games. Having emphasised that it is part of the very concept of winning that winning is what games aim at, Dummett continues,

> Likewise, it is part of the concept of truth that we aim at making true statements; and Frege's theory of truth and falsity as the references of sentences leaves this feature of the concept of truth quite out of account. [4, 50]

We have here a statement of what Priest terms the *teleological theory of truth* [11, 56].[8] An adequate grasp of the concept of truth requires not only that a language user is extensionally correct with respect to the application of the truth predicate but also that she understand the purpose

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[5]On logical inferentialism, see [8] and [5]. In the background is Wittgenstein, 'For a *large* class of cases of employment of the word 'meaning' – though not for all - this word can be explained in this way: the meaning of a word is its use in the language'[20, 43]

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[7]Harmony is not forthcoming on a standard natural deduction treatment. Proposals for recovering harmony appeal either to a multiple conclusion framework [14] or to bilateralism [15]. On the latter see below, but on the specifics of the classical-intuitionist debate here it is important to be aware of [9].

[8]Note that a teleological theory of truth is perfectly compatible with alethic pluralism. Truth may be the *telos* of assertion, but it may be that the kind of reasons statements are correctly assertible differ significantly across different classes of statements.

for which that predicate is employed. Imagine an expressively impoverished language containing only twenty atomic sentences, a truth predicate and no logical connectives. It so happens that every atomic sentence is written on a certain blackboard, and that the true ones are written in green, the false ones in red (this is entirely accidental, there was only a small amount of chalk left and the two colours had to be shared out). A child learns, by observing others, to say 'is true' of all and only the green sentences, let's suppose there are ten of them. She is therefore extensionally correct in her use of the truth predicate. Does it follow that she understands the predicate? No, the Dummettian will say, because she does not have a grasp of the end towards which the truth predicate is deployed, namely indicating those statements at which the practice of assertion aims. Of course, what is meant here is that the *primary* purpose of assertion is the making of true statements. Language users may assert in order to lie, to approximate the truth or simply to say something when confronted with an unexpected question in a viva examination, but all of these purposes are somehow parasitic upon the making of true statements.

More generally, thinks Dummett, in giving a philosophical account of a concept we ought not simply to track use. Rather, 'we must also give an account of the *point* of the concept, explain what we use the word *for*.'[4, 50]. Now, we might ask, what about falsity? By parity with the case of truth, it is not sufficient for a language user's use of the predicate 'is false' to be extensionally correct, as it might be for example if she asserts $\ulcorner P$ is false$\urcorner$ just in case $\ulcorner \neg P \urcorner$ is true. Rather she ought to have an understanding, at least implicit, of the primary purpose for which we attribute falsity to statements. We need to know more about falsity!

Might it be that falsity is the aim of *denial* just as truth is the aim of assertion? Moreover, could there be an interesting relationship between the speech-act of denial and the operation of negation, which does not simply reductively explain the former in terms of the latter and which ties falsity, negation, and denial together in a web of conceptual interconnections? My answer to both questions is positive – of which more below – but first we ought to consider an argument that the application of the Dummetian argument about conceptual grasp to the concept of falsity is illicit, since there is an important disanalogy between the concept of truth and that of falsity.

In arguing for a teleological theory of truth, Priest points to the close connection between truth and meaning. The meaning (Fregean sense) of a declarative sentence is given by its truth-conditions, but if an informative theory of meaning is to be given on a truth-conditional basis there has to be more to our concept of truth than is given by every instance of the T-schema [11, 59-61]. He invites us to consider the schema:

(T) $T\ulcorner\alpha\urcorner \leftrightarrow \alpha'$

and observes that,

> . . .the T-scheme cannot be considered as simultaneously specifying both the sense of α and what it is for α to be true, which is what it would do if the T-scheme were all there is to truth. (T) must be read either as: α is true iff. . . or as: α **is true** iff. . . It cannot be read in both ways at once. [11, 60]

Thankfully, the teleological theory relieves the T-schema of the task of providing a full characterisation of truth, leaving it free to do the heavy-lifting required by a truth-conditional theory of meaning. Now, suppose Priest's argument is correct. It might be thought that it is suggestive of a crucial difference between the concepts of truth and falsity which undermines the case outlined above that a purely extensional characterisation of falsity is insufficient. For *truth* is internally related to *meaning*, and it is this foundational role of the concept of truth in the theory of language, it might be argued, which makes a purely extensional understanding insufficient. Since falsity does not play the same foundational role, we require less of an account of the concept of falsity and can rest content with a purely extensional understanding. So it could be argued.

At least two things may be said in reply. First, and more radically, the supposition that falsity is not salient to meaning could be challenged. If one is committed to the classical behaviour of truth and falsity, as of course Frege was, then it is natural to see truth-conditions alone as meaning constitutive. What I understand when I understand a sentence (grasp its sense) are the conditions under which it is true. I thereby automatically understand the conditions under which it is false, since these obtain just in case its truth-conditions do not obtain. But now suppose that a sentence can fail to be true without thereby being false, or that it can be false whilst also being true. Isn't there some pressure to say that in order to understand a sentence from the relevant class I need to understand its falsity conditions, where these cannot be assumed to be the truth-conditions of its negation? For unless I understand the conditions under which the sentence is false I am not in a position to understand its deductive consequences,[9] and if that is so then I do not understand the sentence itself. For understanding a sentence is knowing how to use it, at least with respect to its canonical use, what Brandom terms 'the giving and receiving of reasons' [3]. Below I will endorse a modification of this view, which takes falsity-conditions to be partly meaning constitutive but denies falsity itself an explanatory role with respect to meaning.

Secondly, the case against extensional adequacy being sufficient for conceptual grasp is a general one, and does not apply solely to concepts with an important theoretical role (such as *truth*). Consider the concept *rabbit*. If I have learned by rote that Flopsy is a rabbit, that Mopsy is a rabbit, that Cottontail is a rabbit, and so on for all rabbits, do I thereby understand 'is a rabbit'? Surely not; for consider a world in which all and only the rabbits are white. On a purely extensional account of the matter there is no difference between an inhabitant of this world's understanding of 'is a rabbit' and her understanding of 'is white'. This can't be right, so conceptual grasp cannot be purely extensional. And so, in particular, our grasp of the concept of falsity cannot be purely extensional.

The Frege-Geach picture doesn't offer us an account which is satisfactory in these terms: on that picture falsity is just to be reductively analysed as the truth of negation, and that is not a sufficient analysis. So we are now in the dialectical situation of needing accounts of both denial and falsity, ideally tracing their inter-relations and the relation of each to negation. Below I will develop such an account, based on a rejectivist understanding of denial and negation and a teleological understanding of falsehood. Before doing that, however, it is worth looking at another position which has un-Fregean things to say about denial and negation, dialetheism. I think the dialetheist's approach to these issues is beset with problems, and attention to the reasons for this will prove instructive.

3. DIALETHIC DENIAL?

A *dialetheist* believes that there is at least one true contradiction, or dialethia. One consequence of this position is that they cannot accept the identification of denial of P with the assertion of $\ulcorner \sim P \urcorner$. For suppose that P is a dialethia, then $\ulcorner \sim P \urcorner$ is true. But one ought not to deny true statements, so P is not deniable [11, 6.2]. Since, however, the dialetheist ought to be prepared to assert $\ulcorner \sim P \urcorner$, asserting this cannot be the same as denying P. More generally there cannot be any sentence the assertion of which is guaranteed to be equivalent to the denial of P for arbitrary P: not only do we have to contend with the situation in which that sentence is itself a dialethia, we also need to take account of trivialism, the doctrine that every proposition is true. Whilst it is undoubtedly unattractive trivialism cannot, thinks Priest, be ruled out as a matter of *logic* (indeed, the paraconsistent logic **LP** has a trivial model). If trivialism holds, no proposition is deniable and every proposition is true. In particular, P is not deniable and the proposition expressed by a candidate sentence for expressing the denial of P is true. Thus that sentence cannot express the denial of P after all.

The dialetheist is best advised to admit a *sui generis* speech-act of denial, and this is indeed the path Priest takes. If she takes this view of denial, the dialetheist has an alternative account to the

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[9]In the sense of recognising a deductive consequence of the sentence when presented to me. Of course we never in practice know all of the deductive consequences of any sentence in a practically adequate language.

Fregean and can moreover explain how dialetheism is compatible with the intelligibility of disagreement – it might look puzzling, on a dialetheic view of things, how we can ever do so much as disagree with someone. Asserting $\ulcorner \sim P \urcorner$ won't do since, according to dialetheism, this is compatible with P, and so we might not want to deny P. Answer, on the Priestian scheme: there is no problem here, we can express disagreement by *denying* P. Add to this the position that P is false just in case $\sim P$ (Priest doesn't want to deny *that*) and the dialetheist has a package deal, accounting for negation, denial and falsity.

What can be said about the interconnection of these concepts? It might seem as though the mutilation of the position sketched above is minimal. For sure the dialetheist cannot allow that falsity is the *telos* of denial, for there are false propositions we ought not to deny, namely dialethias. But she can say that mere falsity is the telos of denial, where a proposition is *merely false* just in case it is false and not also true. She affirms the usual equivalence between falsity and the truth of negation. So it could appear that she has a satisfactory story to tell about these matters. Appearances are deceptive. We started off above wanting to know the purpose of the concept of falsity, for reasons paralleling Dummett's concerning the concept of truth. We have so far heard nothing about this, but only about the concept of mere falsity (we could call this *schmalsity*). But couldn't the dialetheist say that falsity is the telos of denial in the absence of concurrent truth? This hardly accounts for the central role played by falsity in our epistemic practices. Think of it like this: if all that is to be said concerning the point of the concept of falsity is that it is the telos of denial in the absence of concurrent truth, then why not simply ditch the concept of falsity altogether and embrace the concept of schmalsity, which has an obvious point (it is the telos of denial). There is no point in saying that this would be self-defeating for the dialetheist, who by definition wants to claim that some propositions are both true and false. We have yet to be told anything about the point of classifying a proposition as false, whether or not we also classify it as true.[10]

It might be said that the concept of falsity allows us to classify those propositions with true negations. This is true, but it is unclear why that is useful. We can't at this point appeal to the utility of negation in indicating falsehood; this would be arguing in a very tight circle. Priest's account of the nature negation an entirely formal one,

> We see that there appears to be a relationship of a certain kind between pairs such as 'Socrates is mortal' and 'Socrates is not mortal'; and 'Some man is mortal' and 'No man is mortal'. The traditional way of expressing the relationship is that the pairs are *contradictories*, and so we may say that the relationship is that of contradiction. Theories of negation are theories about this relation. [12, 77]

In the section quoted, Priest goes on to note two standard features of the behaviour of contradictory propositions ϕ and $\ulcorner \sim P \urcorner$:

(LEM) $\Box(P \vee \sim P)$

(LNC) $\Box \sim (P \wedge \sim P)$

(Note that LNC is valid in **LP**, contrary to any intuition that it represents a neutral statement of the law of non-contradiction.[11])

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[10]Note especially that thumping the table and saying that the point in classifying a proposition as false is to indicate the belief that it is false *in reality* will achieve absolutely nothing here. We've yet to be told anything about the significance of a proposition being false *in reality*. Compare: 'The point of classifying objects as *grue* is to indicate the belief that they are grue *in reality*'.

[11]*Proof sketch*: Without loss of generality, we need only consider an arbitrary dialethia $(P \wedge \sim P)$. Since P is false, by the truth of $\sim P$, it follows that the conjunction is false. Hence $\sim (P \wedge \sim P)$. Necessitation gets us $\Box \sim (P \wedge \sim P)$ as required. $\Box$ On the law of non-contradiction, see the contributions to [].

This gets us no further unless some reason is given why it is desirable to be able to mark pairs of contradictory propositions. The natural way to do this might seem to be in terms of falsity. But remember the dialetheist has yet to tell us the point of the concept of falsity.

Dialetheism is often dismissed speedily, as absurd or obviously false (and not also true). This is unfair to its proponents. Much has been done in recent decades to supply philosophical motivation and formal articulation for the doctrine, and the results are highly sophisticated [11] [1]. As I see it, the problem with dialetheism is more subtle than those standardly raised in the literature. It leaves two central logical concepts, falsity and negation, floating free in conceptual space, without any adequate account of their purpose in our conceptual repetoire. Meanwhile a *sui generis* speech-act of denial bears no interesting relation to either falsity or negation. Things that intuitively belong together have been divided. In the next section I will put them back together again (in a non-dialetheic context).

4. An alternative picture

Where are we at? The Frege-Geach orthodoxy does not hold up. We are lacking an adequate account of the relationship between denial, negation and falsity. Such an account would exhibit the relationship between these concepts, whilst making lucid their purpose in our practical and rational lives. The purpose of this section is to supply such an account. My proposed account is rejectivist, that is I explain negation in terms of denial, rather than vice versa. I will develop this rejectivism through a consideration of Ian Rumfitt's bilateral proof theory, before going on to show how a teleological account of falsity sits naturally alongside this account.

4.0.1. *Bilateralism.* On a logical inferentialist understanding, which I will be taking to constrain the present dialectical context, the rules of deductive use are constitutive of the meaning of logical expressions. What are those rules? Assuming for present purposes classical logic (nothing much ultimately turns on this), there is an easy, obvious, answer: the rules are those present in a standard natural deduction system, such as that presented in []. These rules are constitutive of meaning, and normative for reasoning. There is a lot that is correct here, but there is a curious feature of the set-up. We not only assert statements; we also deny them. And whilst, compatibly with the rejection of the Frege-Geach account, we might allow that speakers sometimes express denial through uttering the corresponding negation, this is certainly not the only means available to them. They might shake their head, (as Rumfitt points out) simply say the word 'no' in response to a question, or do any number of other things. Standard natural deduction systems are *univalent*, they detail conditions under which logically complex statements may be asserted, but do not provide conditions for any other speech-act. Given what we have just noted about denial, however, this looks like an arbitrary curtailment of logical concern. For we not only assert, we also deny, and deductive reasoning is integral to our coming to see what we ought to deny in appropriate circumstances.[12] If I deny P, unless I subsequently retract that denial I ought to deny $(P \wedge Q)$. I cannot consistently deny $(P \vee Q)$ and go on to assert Q.

That denials are integral and basic moves in the game of deductive reasoning is by itself sufficient reason to be interested in formalising denial.[13] Rumfitt develops a bilateral proof theory, providing conditions for both the assertion and denial of logically complex statements, with another motive in view. Dummett has argued that the debate between realism and anti-realism about many philosophically significant domains of discourse maps onto, and may be decided in terms of, the debate over whether classical or intuitionist logic is the correct logic for reasoning about the relevant domain [] []. This opens the door to an argument for *global* anti-realism which runs (roughly) as follows: it is

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[12]'In appropriate circumstances' here is intended to gesture towards debates about the normativity of logic []. My view is that a proper recognition of the social nature of reasoning, which sits comfortably with the perspective on logic as rule-governed presented here, dissolves much of this dispute. For a similar view see [10], but developing this position requires further work elsewhere.

[13]'Game' here is intended in the sense of Brandom's 'game of giving and receiving reasons' and Wittgenstein's 'language game' [2][20]. There is no suggestion of unimportance.

a constraint on an adequate set of rules for the logical connectives that they be harmonious; harmonious rules can be supplied for intuitionistic, but not classical, connectives, so intuitionistic logic is the correct logic for reasoning regardless of subject matter. Rumfitt replies on behalf of the classical logician (and therefore the opponent of global anti-realism.)[14] Harmonious rules can be provided for classical logic, just do long as those rules determine not only the conditions for the correct assertion of their conclusions but also those for their correct denial. A proof system consisting of rules for denial as well as for assertion is described as *bivalent*, as distinct from *univalent*. Rumfitt holds that realist/ anti-realist debates concerning a certain domain are to be adjudicated on the basis whether it is appropriate to provide a bivalent account of the connectives as used to reason about that domain.

Regardless of one's attitude concerning the importance of realism/ anti-realism debates and whether or not Rumfitt succeeds in moving these debates forward, bilateral formulations of proof theory are of independent interest. Above I sketched a rejectivist view of negation: negation is to be explained in terms of the speech-act of denial, rather than *vice-versa*. Bilateralism gives formal expression to just such a view. Atomic statements[15] are equipped with denial conditions, as well as assertion conditions (and grasping the sense of such a statement is constituted by an understanding of its denial, as well as its assertion, conditions). A language-user is entitled to assert $\ulcorner \sim P \urcorner$ just in case she is entitled to deny P, that is: just in case the denial conditions for P are satisfied. In Rumfitt's formulation, the rule for introducing an assertion of a negated formula is:

$$\frac{-P}{+(\sim P)} \; \text{(+Neg-I)}$$

In words, the denial of P is warrant for the assertion of $\sim P$. Since the satisfaction of the denial conditions for P is warrant for the denial of P, we may liberalise our reading of '-' and say that the satisfaction of the denial conditions for P is warrant for the assertion of $\sim P$. It is not necessary for her being in a position to assert $\sim P$ that a reasoner have already, in the course of an argument, have denied that P, but only that she be in a position to deny that P. Negation provides a way of expressing deniability in an assertoric context. There is a clear sense, moreover, in which denial is prior to negation on a bilateral picture. Given that the canonical bilateral introduction and elimination rules express the understanding of a competent user of negation, I cannot so much as understand negation unless I have at least an implicit grasp of denial.

4.0.2. *Falsity and primitive denial.* What, though, of falsity? This was a cause of difficulty earlier on. Following Dummett's work on truth, I insisted that a purely extensional characterisation of falsity cannot be adequate. In particular, whilst it is true that P is false just in case $\ulcorner \sim P \urcorner$ is true, recognition of this does not suffice for a grasp of the concept of falsity. This requires an understanding of the purpose for which the concept is deployed. Once we take denial seriously, we can see how language users can obtain such an understanding. Like truth, falsity, is to be understood in *teleological* terms. Falsity is the end of denial. The false statements are those that, if an agent is playing the game of deductive reasoning perfectly, she will (presented with the opportunity) deny. Falsity is anchored in a rule-governed human practice,[16] reasoning, and the falsity conditions of statements can be analysed with no remainder in terms of denial conditions.

A worry expresses itself as the doubt that we are any further ahead than when we started. We were unsatisfied with primitive falsity, and have replaced it instead with a primitive speech-act of denial. Falsity is what denial aims at. Denial it seems is, as Wittgenstein said of language-games, ' there, like our life' [19, §559]. But what advantage, then, does our account have over one based

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[14]Which isn't to say that Rumfitt affirms global realism. His view is that realism/ anti-realism disputes ought to be approached as concerning whether a univalent or a bivalent proof theoretic semantics is appropriate for the disputed domain. See [9] for doubts about this.

[15]On *statements* as the relata of the consequence relation, see [16, 22].

[16]Those who find the framework of grounding insightful might want to appeal to it here: facts about falsity are grounded by facts about the world plus facts about the practice of denial (part of the larger practice of reasoning). I do not endorse that framework (for cases against it, see [18] and [17]), but it is there for those with more stomach!

on primitive falsity? Any philosophical treatment of the trio of concepts looks likely to need at least one primitive, so it is perhaps no weakness of our account that denial bears the weight, but it might be thought equally that there is no basis for preferring our account to an alternative for which falsity (or for that matter, negation) is primitive.

On the contrary, recognising denial as primitive situates falsity and negation in a human practice, denying. It explains their purpose for our practical lives, and explains the role that falsity and negation play for creatures such as ourselves. This takes for granted that we are in fact the kind of creatures who deny things, but that is an appropriate thing for philosophy to take for granted. If there is a question about why we are in the habit of denying, or why we find it useful to deny, then that is a matter for anthropology, not philosophy. Philosophy can rest content when it has been shown that logic's trio of negative concepts have a basis in our practical life, and the connection between them elucidated. That has been done. *

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