ON PLURAL ROBUSTNESS

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ABSTRACT. Plural Robustness is the claim that plural quantifiers are more determinate than their set-theoretically interpreted second-order equivalents. This principle plays an important role in arguments about the use of plural logic in the philosophy of mathematics. Recent work by Florio and Linnebo develops a nonstandard semantics for plural logics, which is used to challenge Plural Robustness. We discuss the motivation for Robustness, examining the use of meta-linguistic plurals to interpret plural expressions in the object language, and suggesting that the challenge to this should be understood as a form of semantic scepticism. We then argue that in a context of set-theoretic pluralism a degree of Plural Robustness obtains. Nonetheless, it is only through attention to foundational metaphysical and semantic questions that progress will be achieved in this debate.

Contemporary philosophical interest in the logic of plurals has its roots in George Boolos' desire to express set-theoretic principles, such as Separation, as single axioms, rather than schemata[1] (Boolos, 1998e) (Boolos, 1998c). Using the now standard terminology, where '*xx*' is a plural variable and '$\prec$' reads 'is amongst', we can state Separation:[2]

(SEP) $$\forall xx \forall x \exists y \forall z\, (z \in y \leftrightarrow (z \in x \wedge x \prec xx))$$

Similarly, we can afford ourselves a finite axiomatisation of arithmetic, stating induction as a single axiom:

(IND) $$\forall xx\, ((0 \prec xx \wedge \forall n(n \prec xx \rightarrow s(n) \prec xx)) \rightarrow \forall n\, n \prec xx)$$

A natural question about the mathematical theories characterised by principles such as these is whether, in addition to the gain in expressive power present in (SEP) and (IND) by contrast to first-order schemata, they possess the capacity to characterise the target mathematical structures more

[1]Boolos himself used monadic second-order logic, interpreted with natural language plurals, for this purpose. We will use a distinctively plural language after the fashion of (Rayo, 2007).

[2]With Separation stated like this, Empty Set needs to be added as an axiom. The iterative conception of sets surely supplies motivation for this addition. Functional quantification, needed for the statement of replacement, can be implemented by plural quantification over tuples.

satisfactorily than do their singular first-order equivalents. Several authors on plural logic have answered this question in the affirmative (Hossack, 2000) (Yi, 1999). For these authors plural logic, like second-order logic with the standard semantics, affords us categorical theories for arithmetic and analysis, and quasi-categorical set theory.

This is too quick, however. In the case of second-order logic with set-based semantics, we are familiar with the availability of non-standard, Henkin, semantics to which the limitative results characteristic of first-order logic (completeness, compactness, and the Löwenheim-Skolem theorems) apply. Florio and Linnebo have shown that a parallel non-standard plurality-based semantics can be provided for the language of plural logic (Florio and Linnebo, 2015).[3] In the light of this, we may wonder whether the proponent of plural logic is in any better position than her colleague who champions second-order logic, at least as regards the provision of structure-characterising mathematical theories. If there is a question about the semantic determinacy of second-order quantifiers, arising out of the availability of non-standard semantics, does not exactly the same question arise about the semantic determinacy of *plural* quantifiers, and for exactly the same reason?[4]

The view that plural logic stands up better to the worries about indeterminacy than does its second-order cousin is termed *Plural Robustness* by Florio and Linnebo(Florio and Linnebo, 2015). Regardless of whether Plural Robustness holds plural logic enjoys *some* advantages over second-order logic. There are applications to which the former, but not the latter is suited. A clear example is the formalisation of set theory, on the intended interpretation in which the first-order variables range over all sets. If second-order logic is used for this purpose then,

$$\exists X \forall x\, Xx \leftrightarrow x \notin x \tag{1}$$

commits us (in the metatheory) to the existence of the Russell set. However (1) is an instance of second-order comprehension and is therefore valid in second-order logic. For this reason Boolos, early in his investigation of higher-order logic, thought that second-order quantification should be

[3] Hereafter I will use ‘second-order logic’ in a sense that implies the provision of a set-theoretic semantics and ‘plural logic’ in a sense that implies the provision of a plurality-based semantics. ‘Higher-order logic’ is intended in a sense that encompasses both plural and second-order logics.

[4] This question might be thought of as being directed both to formalised plural quantification and to the natural language reasoning that this is generally thought to formalise. Although see (Ben-Yami, 2009) for scepticism about the existence of natural language plural quantifiers.

avoided in the formalisation of set theory (Boolos, 1998d). Yet, as he was later to realise, the difficulty can be avoided by a move to plural quantification. The plural variant of (1),

$$(2) \qquad \exists xx \forall x \, x \prec xx \leftrightarrow x \notin x$$

avoids the problematic commitment, since the witnesses for the plural existential quantifier are simply the non-selfmembered sets,[5] not the Russell Set. This genuine benefit of plural logic is likely to be missed if either one is sceptical about the possibility of quantification over all sets or one thinks that every plurality determines a set.[6] Both of these positions are substantial philosophical claims, and unless one is persuaded of at least one of them there is good reason to regard plural logic as having advantages over second-order logic regardless of whether Plural Robustness can be supported.

That said, it is undoubtedly true that if Plural Robustness is unsustainable then many of the hopes invested in plural logic by philosophers are in vain. The purpose of the present paper is to offer a partial defence Plural Robustness, and so vindicate at least some of these hopes. We proceed as follows. Section One lays out Florio and Linnebo's arguments against Plural Robustness. Section Two addresses one strand of argument by raising the prospect of set-theoretic pluralism, which supports Plural Robustness in some cases but not others. Section Three argues against the legitimacy of the move Florio and Linnebo seek to make from plural determinacy to the determinacy of set-theoretic semantics for second-order logic. Section Four reflects on the implications of the partial result regarding Plural Robustness.

The explicit conclusion of this paper is that, since set-theoretic pluralism is presently an epistemic possibility, Florio and Linnebo's absolute rejection of Plural Robustness is not supported. Conversely, it is only in some, but not all, theoretical contexts that Plural Robustness can be defended. Not fully explicit anywhere but the present paragraph, but implicit throughout is the conviction that in order to make progress with the debate about Robustness we will need to resolve the question whether set-theoretic pluralism is true. This, in turn, will require attention to fundamental semantic and metaphysical questions about mathematics. This is as it should be. For whatever else logic might be, it is a toolkit for reasoning about reality. It is scarcely surprising that questions about the

[5]Assuming that Foundation is unrestrictedly true of sets, these are just the sets.

[6]Linnebo's work on the philosophy of set theory combines both intuitions (Linnebo, 2010) and (Linnebo, 2013). For a recent alternative approach to advocating the indefinite extensibility of sets see (Uzquiano, 2015). The 'naive intuition' that every plurality determines a set is opposed in (Hewitt, 2015)

relative merits of various of the tools in the box turn on the nature of that reality and of our access to it in language and thought.

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Florio and Linnebo have established beyond doubt that there are plurality-based yet nonstandard interpretations of plural languages. For the technical details, readers are referred to their paper. What are the implications of this for Plural Robustness? At the very least it places on the table,

> **Parity claim (P):** Second-order logic with set-based semantics and plural logic with a plurality-based semantics are on a par as regards semantic determinacy.[7]

Florio and Linnebo defend **(P)**. They first address a common argument for the determinacy of plural quantifiers which proceeds from the assumption of *plural innocence* – the view that plural quantification incurs no ontological commitments over and above those incurred by singular first-order quantification over the same domain. The argument is articulated by Keith Hossack,

> The singularist[8] cannot solve the problem of indeterminacy, but the pluralist can. . .. Plural set theory has no non-standard models, so the indeterminacy problem does not arise for pluralism, . . . plural variables range plurally over the very same particulars that the singular variables range over individually. Therefore the pluralist does not confront an independent problem of identifying what the plural variables range over. . . Plural sentences therefore provide the missing additional constraint we were seeking on admissible interpretations. This is why the pluralist is able to solve the indeterminacy problem, though the singularist cannot do so. (Hossack, 2000, 440-41)

For Hossack, then, once the first-order domain is given the range of the plural variables is an immediate corollary. Since there are no plural entities – plural variables simply range over entities *in plurality* – there is no separate plural domain, rather plural variables range over the singular first-order domain in a distinctive, which is to say plural, fashion. In outline the argument is:

(1) Plural variables range over the same domain as singular variables.

(2) So, given a singular first-order domain, there is no further question of a plural domain.

[7]Florio and Linnebo's statement of the claim talks about *plural* logic with a set-based semantics.

[8]A singularist, for Hossack, is someone who only admits singular quantification. Her adversary is a *pluralist*. This usage varies subtlely from other parts of the literature, such as (Florio, 2014).

(3) So, plural quantification is determinate if the corresponding singular first-order quantification is determinate.[9]

Implicit is the gloss on (1) and (2) whereby the plural variables range over every possible combination of singular entities on the domain, of which there are 2^κ for a domain of cardinality κ, yielding the desired metalogical properties of plural logic.[10] (1) is motivated by plural innocence: since plural quantification should not be understood as singular quantification over sets, fusions, properties, or whatever, but rather as genuinely plural quantification, no question arises about a separate plural domain. There is simply one domain, the familiar old domain ranged over by the singular variables. To be is, as it has always been, to be the value of one of these.[11]

Florio and Linnebo's response to Hossack is best understood as questioning whether plural innocence suffices to motivate (1). They draw attention to the fact that their Henkin-style semantics is just as innocent as the semantics favoured by Hossack since, on both, 'plural variables range plurally over objects in the ordinary, first-order domain' (Florio and Linnebo, 2015, 7-8).[12] This much is certainly true. I doubt, however, that it addresses the full motivational impact of plural innocence with respect to (1). For this consists not solely in the suggestion that any restriction on the range of plural variables, relative to the combinatorial possibilities on the singular domain, would violate plural innocence: when by innocence and *modus tollens*, (1). Florio and Linnebo's provision of a Henkin-style semantics which assigns pluralities to the plural variables parries *this* line of attack. Instead, given plural innocence, there is an issue about the *meaning* of plural quantifiers: if we are not to interpret them as singular quantifiers over some kind of collectivising entities, but rather as *sui generis* plural quantifiers, we are confronted with a question, 'yes, but *which* pluralities are being quantified over?' And now the contention is a simple one: *all* of them.

[9]Considerations about vagueness or, pertinently in the philosophy of mathematics, indefinite extensibility and/ or constructivism might feature in the denial of the antecedent here. However, all parties to the debate in the current literature seem content to concede singular first-order determinacy.

[10]If the domain is finite (as it isn't in the cases we're interested in), there will in fact be $2^\kappa - 1$ pluralities on the domain, since there is no empty plurality (there are no things such that nothing is one of those things). A familiar workaround owing to Boolos allows plural theories to stand in for their second-order cousins in spite of the background logical theories differing regarding the empty case.

[11]This observation is not in tension with the title of Boolos' 'To be is to be the value of a variable (or some values of some variables)'(Boolos, 1998e) : take some values of some variables under some assignment, each one of them individually is already the value of a singular variable under some assignment, and thus is already included in the catalogue of being. To propose a broader notion of ontological commitment which counts true plural existential quantifications as carrying additional commitments – after the example of (Rayo, 2002) – seems to me to be at least one of double counting rooted in a failure to take seriously the distinctively plural nature of plural quantification.

[12]Florio and Linnebo in fact doubt plural innocence, but are allowing it here for the sake of argument. Addressing their concerns on plural innocence is work for elsewhere.

How does this move – 'all' means *all* – relate to innocence? First, as we shall see in due course, for a key case of ontologically committing higher-order quantification, set-theoretically interpreted second-order quantification, there might be thought to be a genuine ambiguity about which sets are *all* of the sets. Innocence does away with a parallel worry in the plural case. Or does it? The attentive reader will by now be complaining that when asserting in the previous paragraph that (1) is secured by quantification being over *all* the pluralities, I was assuming the determinacy of plural quantification in the metalanguage, namely the philosopher's English in which this paper is written. This is true: if we want to admit that plurals are a genuine semantic phenomenon,[13] then there is no more hope of avoiding the use of plural metatheoretic resources in spelling out the intended interpretation of a plural object language than there is of avoiding using conjunction in stating a perspicuous semantic clause for '$\wedge$'.[14]

It is possible of course to defend the determinacy of the metatheoretic plural quantifier by ascent to the meta-metatheory and repeat of the "'all" means all' move. At this point, our interlocutor will repeat her objection, and we are off on a sceptical regress. Generalising for any plural language L_α, we can argue for determinacy in $L_{\alpha+1}$. It is perfectly possible, on a sufficiently abstract and mathematicised understanding of languages, to imagine this process being extended to a limit,

$$\beta = \bigcup_{\alpha<\beta} \alpha$$

such that L_β possesses the resources to settle determinacy for all languages indexed by a smaller ordinal[15]. But then, the response comes, what about the determinacy of the plural quantifiers in L_β? And we're off again: 'that is settled in $L_{\beta+1}$'. In reality, absent the provocations of the sceptic, we will stop the ascent much earlier. In fact, for usual purposes, it is at the level of the first metalanguage that we, in Quine's phrase, 'acquiesce in our mother tongue': interpreting the quantifiers of our formal language using antecedently understood natural language plurals. We cannot step outside of language and gain a God's eye perspective on semantics. There is never any prospect of converting a sceptic, and it would be foolhardy to attempt to do so, following her off up the regress in a doomed attempt to block her every manovure. We should set our sights more modestly. In what follows I will argue that potential problems for the determinacy of second-order logic do not arise for plural

[13]The contrast here is with what Florio terms 'semantic singularism' (Florio, 2014).

[14]One could of course give a clause like $V(\phi \wedge \psi) = \text{Min}[\phi, \psi]$, with 1 truth and 0 falsity. But even if this genuinely avoids the use of conjunction (there's surely a temptation to read $\ulcorner\text{Min}[\phi, \psi]\urcorner$ as $\ulcorner$the least value had by ϕ and $\psi\urcorner$) it hardly discloses the meaning of conjunction to someone who does not already grasp it.

[15]For similar considerations, see (Linnebo and Rayo, 2012)

logic, and that there is reason to believe that a determinate and maximal meaning is the intended meaning of the plural quantifier in relevant contexts. Or, more carefully, I will argue this conditional upon metaphysical and semantic assumptions which Florio and Linnebo give no impression of not sharing, and which seem to be implicit in the classical mathematics taken for granted in the authors' other work. That is enough reassurance of robustness for the plural logician to be getting on with.

Before we lay arguments focused on innocence to rest, and in anticipation of the case about intended meaning, it deserves note that whilst the Henkin-style semantics assigns pluralities to plural variables, and so conforms with plural innocence, the semantic clause for variable assignments makes non-eliminable appeal to a plural property. The function of this property is to specify a domain for the plural variables: xx are only available to be assigned to a variable if $\mathbf{D}(xx)$, where $\mathbf{D}$ is the plural property and relates to the singular domain[16] dd thus:

$$\forall xx(\mathbf{D}(xx) \rightarrow \forall x(x \prec xx \rightarrow x \prec dd)$$

Florio and Linnebo anticipate resistance at this point, on the basis that the appeal to plural properties violates innocence. Such an objection would be somewhat misplaced, since the plural property is not assigned to any item of the object language as its semantic value. For sure, our semantic theory is committed to plural properties, but there is good reason to believe in these in any case (Hossack, 2000) (Yi, 1999). In the spirit of a certain current in contemporary metaphysics, it might be objected that belief in *some* plural properties is one thing, but that our theory of plural properties, like our theory of properties in general ought to be sparse and not admit the kind of arbitrary plural properties required to generate enough models for a Henkin-style semantics (Lewis, 1997). Here Florio and Linnebo minute a possible recourse to *superplurals* rather than plural properties, but the legitimacy of these is also controversial.[17]

However, it is not the commitments, whether ontological or ideological, of the metatheory that most warrants scrutiny when assessing the role of the Henkin-style semantics in motivating **(P)**. Rather, the *rôle* of $\mathbf{D}$ speaks against the interpretation of plural quantifiers according to this semantics being the intended one. The function of $\mathbf{D}$ is to *restrict* the domain of plural quantification in all but one model for any given dd and interpretation function.[18] There *is* an unrestricted and maximal

[16] The singular domain is specified in the metatheory as a plurality, rather than as a set.
[17] On superplurals, see (Rayo, 2006)
[18] As with the set theoretic semantics, there is a limiting case of a faithful Henkin model equivalent to the standard model.

domain, from the point of view of the metatheory, and plural reference can be made to it – as with '*dd*' – and plural variables range over it, as with the last above quoted formula. From this perspective, then, Henkin models for which $\exists xx \sim \mathbf{D}(xx)$ capture a deviant meaning for '$\forall$'. This is not to dismiss such models out of hand; the logician may make use of them as she wishes. But what they do not do is encode the meaning of the quantifiers, since it is a constraint on this that '$\forall$' means all (and since the quantifiers are classically indefinable, the determinacy of '$\exists$' follows by corollary).

In fact, we can say more than this: the position of the philosopher who holds that plural quantifiers are either indeterminate or have a determinate, but non-maximal, interpretation is self-undermining. For this philosopher cannot, by her own lights, describe a semantic theory which adequately captures the meaning of plural quantifiers (as she understands them), such that the disagreement between her and the philosopher who holds that plural quantifiers have a determinate and maximal meaning is genuine. In order to do this, she would need to be able to express the thought that her plural quantifiers have a non-maximal range: but she cannot do this. Either the plural quantifiers in her metalanguage are determinate and maximal, or they are not. If they are not, then she transgresses her own position, much like David Lewis' imagined opponent of absolutely general quantification who 'violates his own stricture in the very act of proclaiming it' (Lewis, 1991, 68). If they are restricted, then she cannot express disagreement with her opponent, and the debate dissolves. Either way, the position is self-refuting.[19]

This line of argument may support a determinate and maximal interpretation of plural quantifiers, but does it provide reason to accept Plural Robustness? After all exactly the same considerations can be summoned to dismiss Henkin semantics for second-order logic as unintended. For a given domain $\mathcal{D}$, the Henkin domain for (monadic) second-order variables is $\mathcal{P}(\mathcal{D}) \setminus A$ for $A \neq \emptyset$ in all but the standard case. So, as before, from the perspective of the set-theoretic metatheory nonstandard models can be viewed as representing restrictions on the quantifiers that are unwarranted to the extent that '$\forall$' is supposed to mean *all*. This is correct: if the argument to this point were all that there is to be said, we would have reason to be assured about the determinacy of plural quantification but not about Plural Robustness. It is just as well, then, that there is more to be said.

[19]Which is what we should expect, if I am correct in understanding the issue here as being about a version of semantic scepticism. In fact, the dialectic of the proceeding paragraph is recognisably similar to that of a certain kind of response to a skolemite sceptic about the uncountably infinite. On this, see (Button, 2016).

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It is well known that there are propositions independent of (first-order[20]) ZF. These include the Axiom of Choice, the Continnum Hypothesis (CH) and Generalised Continuum Hypothesis (GCH) and the Suslin hypothesis. All but the first (unsurprisingly!) are independent of ZFC. An increasingly common thought in the philosophy of set-theory is that the failure of our best theory to settle these open questions points to a more fundamental indeterminacy in set-theory.[21] Might it not be the case that, rather than there being a single set theoretic universe, over which it is determinately the case that the variables bound by set theoretic quantifiers range, there are in fact a multiplicity of such universes? Since the advent of inner models and forcing, set-theorists have been used to working within a wide variety of models satisfying diverse sentences. They feel at home with such models, many of which give rise to fruitful and interesting mathematics. Is it not more natural to regard all of these models, or at least more than one of these models, with metaphysical seriousness, as each corresponding to a universe within a multiverse which is home to set-theoretic mathematics (Hamkins, 2012)? This multiverse view is just one form of a metaphysical pluralism about set-theory which is growing in popularity (Steel, 2014) (Arrigoni and Friedman, 2013).[22]

Set-theoretic pluralism (STP) is not without opponents, and it is not the purpose of the present paper to assess whether some form of it is correct.[23] However, the view has a significant number of proponents, and may therefore be considered as posing an open question so far as current philosophy is concerned. Suppose that STP is correct: what does this imply about Plural Robustness? In what follows we will disregard the extreme form of STP advocated by Hamkins and restrict our attention to universes corresponding to well-founded and natural models, or at least ω-models, of set-theory. This is not an unprincipled restriction; the motivation is the recognition that, whatever else may be the case, universes of set-theory serve as foundations for mathematics. This being so, we have good reason to take them to be constrained by conformity to our settled beliefs about mathematical reality (and, in particular, about the structure studied by arithmetic). The ontology of set-theory, in other

[20]The situation with second-order ZF is discussed in Chapters 5 and 6 of (Shapiro, 1991)

[21]In the discussion that follows, I assume that set-theoretic pluralism might be extended to set-theory with *urelemente* (this is important, since we will want to consider sets of natural numbers, for example, without assuming that natural numbers are to be identified with certain sets). It is important dialectically that the set-theory is taken to be first-order, or at least, if second-order, that the *urelemente* are not assumed to form a set, owing to McGee's categoricity result (McGee, 1997).

[22]The lexicographic relationship between 'pluralism' and 'plurals' is unfortunate and potentially confusing, but unavoidable in the light of the literature.

[23]For negative answers see (Martin, 2001) and (Woodin, 2012).

words, has to cohere with the canonical applications of set-theory.

Then second-order categoricity results establish less than is customarily supposed. As presented by champions of second-order logic, these are supposed to reassure us that mathematical theories, as formalised using second-order quantifiers, characterise unique structures, at least up to isomorphism. But the genuineness of this reassurance rests on the determinacy of the set-theoretic metatheory in terms of which semantics are being supplied for the second-order language.[24] To see this consider the induction axiom which characterises PA2:

(2-IND) $$\forall X((X0 \wedge \forall n(Xn \rightarrow Xs(n))) \rightarrow \forall n\, Xn)$$

What is the range of the second-order variables here? Answer: the full classical power-set of the set over which the first-order variables range. That is, in the present case $\mathcal{P}(\mathbb{N})$ – we're assuming here, in the spirit of appeals to categoricity result that any set equicardinal with the 'proper' naturals (if such there be) will 'do' as $\mathbb{N}$. But *which* set *is* $\mathcal{P}(\mathbb{N})$? Or, to push the question in the aforementioned spirit: what is the *cardinality* of this set? This is precisely the most prominent question on which models of ZFC which differ regarding width disagree. It is the question of the size of the continuum, and is radically indeterminate over models. In fact, it is consistent with ZFC that $2^{\aleph_0} = \kappa$ for any κ with uncountable cofinality. If every consistent solution to the continuum problem is instanced in some existing universe, and if it is indeterminate within which universe second-order variables range, then the categoricity result for PA2 does *not* establish that there is a natural number structure unique up to isomorphism. Instead, all that follows is that within each universe there is a natural number structure unique up to isomorphism.

If we allow that the determinacy of second-order quantifiers is undermined by STP, does it follow that a similar indeterminacy afflicts plural quantifiers? It would appear not, unless one is tacitly assuming that plural quantifiers are somehow set-theoretic in nature. For sure, if plural quantification over a domain D is just a re-description of quantification over the elements of $\mathcal{P}(D)$, then indeterminacy will carry across from the set-theoretic case to the plural. But there are good reasons not to regard plural quantification as implicitly set-theoretic. So an argument is required from the opponent of Robustness in a context of set-theoretic pluralism. We'll consider such an argument in

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[24] On this point, see (Meadows, 2013).

what remains of this section.

To understand the argument against Robustness given STP, consider by way of contrast a naive argument *in favour* of Robustness. A set is something over and above its elements – this is the import of Lewis' metaphor of a 'lasso' around the elements – and because of this it is not difficult to imagine there being several non-equivalent ways of forming sets from a given domain. In contrast to this, pluralities just *are* various amongst the elements of the domain. How could there fail to be a determinate fact of the matter which pluralities there are on a given domain? Once you have the domain, it is surely settled which pluralities there are – every way of collecting elements together yields a plurality, and no more is required of the world than this for plural quantifiers to be determinate (contrast set-theoretic quantifiers, where more is required, namely that certain sets exist.) There are 2^{κ} pluralities on an infinite domain of cardinality κ, these are all the pluralities there are, and plural quantification – on the intended interpretation of the quantifiers – is over all of these.[25]

There is a lot to be said for this naive argument. However, it might appear vulnerable to attack at the point of appealing to there being a fact of the matter which pluralities there are on an infinite domain. In particular, notice that we claimed that there are 2^{κ} such pluralities. But what does this mean? Remember that we are in a setting of STP; doesn't this threaten plural determinacy at precisely this point? For the size of 2^{κ} for transfinite κ just is the generalised continuum problem. So if we are correct in thinking there are this many pluralities on a transfinite domain, which seems to be established by elementary combinatoric argument (consider binary strings of length $\kappa \ldots$), then the determinacy of plural quantifiers is surely infected with any indeterminacy accruing to set-theoretic quantification. After all, which cardinality is 2^{κ} is a set theoretic question. Indeed, this entanglement of plural quantification with set-theory should not surprise us, since the question of which pluralities there are on a given domain is a combinatorial question, and set-theory is – amongst other things – the science of infinitary combinatorics.

[25] I anticipate an objection at this stage. If pluralities can be *numbered* then surely they are *objects*, contravening the claim that plural quantification is innocent of distinctive ontological commitments. A first thought is that we might avoid this objection by appeal to coding techniques (let a member of each plurality stand in for that plurality, if necessary using pairs to stand in for pluralities). Unfortunately, this recourse blocks the availability of absolutely general plural quantification on pain of Cantor's paradox. Therefore another response is needed if this objection is felt to be forceful (that is, if the background account of cardinal number is found plausible). One way of doing this might be modal: *if there were more things than there actually are, then there would be a coding such that*…. Alternatively, we could retreat from cashing out maximality in cardinality terms. But if anything this last resort would *help* the position of this paper, since it makes less appealing the thought that assertions of plural maximality are set theoretically entangled. The stance of the main body of the paper is, then, dialectically favourable to my opponent.

Rewind a little. There are three questions which need to be clearly disambiguated:

- The **metaphysical** question: whether it is determinate which pluralities there are on a given domain.
- The **semantic** question: whether plural quantifiers have a determinate range.[26]
- The **epistemological** question: whether I can know which pluralities are within the range of the plural variables of my language.[27]

The naive argument moves from an affirmative answer to the metaphysical question, based on a conceptual analysis of *plurality* (a plurality just is some things), to a similarly affirmative answer to the semantic question, since 'all' means all. At no point is any answer to the epistemological question appealed to, nor is it obvious that this question is at all relevant to the resolution of semantic determinacy. One reason for taking the contrary view would be an anti-realism that grounded an answer to the metaphysical question in one to the epistemological, but this is a significant philosophical doctrine requiring independent motivation.[28] To my mind, it is fundamental debates of this kind that are the proper focus of philosophical discussion of the logic of quantification.

On the face of it, then, our simply not knowing which cardinality is 2^κ doesn't threaten the claim that plural variables on a domain of cardinality κ range determinately over that many pluralities. To suppose otherwise is to confuse an epistemological question with a metaphysical one from which, barring philosophical argument to the contrary, it should be kept separate. But isn't there a metaphysical question waiting in the wings? For, it might be argued, *which* cardinality is 2^κ is a question whose answer is grounded in set theoretic ontology. If there is no fact of the matter which set is the power set of a given set of cardinality κ, with a variety of equally good candidates disagreeing regarding width, then there simply is no fact of the matter regarding the value of 2^κ and consequently no fact of the matter whether plural quantifiers range over this many pluralities. Semantic indeterminacy follows, and Robustness falls.

[26] I take this to be a necessary, and possibly sufficient, condition for them having a determinate *meaning*. As is usual, talk of the range of quantifiers is shorthand for the range of variables bound by those quantifiers.

[27] An affirmative answer to this question will have to be more informative than a blunt 'all of them'. A plausible criterion might be the existence of an effective rule for generating all the pluralities within the range of the variables, but this makes explicit the incompatibility of an affirmative answer with standard-style semantics on a transfinite domain.

[28] Many of the animadversions about quantification over arbitrary sub-pluralities on an infinite domain expressed in the literature seem to me to be favourable to an anti-realism of this sort, without owning it explicitly or following through its logico-mathematical implications. See, for instance, Linnebo's argument in *Plural Quantification Exposed* (Linnebo, 2003). It seems to me that the move from epistemological to metaphysical anti-realism is suggestive of a constructivism about the entities in the domain. If this is correct then questions could be raised about the objectors' entitlement to classical logic.

The argument that set theoretic indeterminacy forces plural indeterminacy, since there is no fact of the matter concerning which pluralities variables bound by unrestricted plural quantifiers range over, can be resisted in one crucial case. This wins us *some* robustness. Consider two models of set theory, $\mathcal{M}_1$ and $\mathcal{M}_2$, with the same ordinals, such that M_1 is thinner than M_2. Now $\mathcal{P}(\omega)^{M1} \neq \mathcal{P}(\omega)^{M2}$. A natural pluralist gloss on this is that there are some pluralities of natural numbers that do not form a set in M_1. Moreover, whilst indeterminacy afflict $P(\omega)$ (we can always force new subsets of ω, no such indeterminacy afflicts ω itself.[29] There is, in the envisaged situation, no question about which objects are the natural numbers. *Ipso facto* there is no question about whether some objects are some natural numbers. So plural quantification over ω is semantically determinate, and plural arithmetic categorical.[30]

Assurance of a categorical plural arithmetic is no small thing. Can we hope for more Robustness than this? Here's one line of argument which, although it will prove unsuccessful, is instructive. Assuming the multiverse picture of set theory, say that a universe is *plurally maximal* for V_α iff there is no plurality of elements of V_α xx such that no element of $\mathcal{P}(V_\alpha)$ is a set of all and only the xx but there is a universe at which $\mathcal{P}(V_\alpha)$ form a set. We can formalise this modally, where '$x \equiv xx$' reads 'x is the set of all and only xx' and where the intended interpretation of the modal operators is such that every universe is accessible from every other universe, and accessibility is reflexive:

Definition.

(3) $$\mathbf{Max}V_\alpha \leftrightarrow_{df} \sim \exists xx\, (\forall x \prec xx[x \in V_\alpha \wedge \sim \exists y[y \in \mathcal{P}(V_\alpha) \wedge \forall z(z \in y \leftrightarrow z \prec xx) \wedge \Diamond(y \in \mathcal{P}(V_\alpha) \wedge \forall z(z \in y \leftrightarrow z \prec xx))])$$

The plural logician now might suppose herself to have a non-arbitrary answer to questions concerning the range of plural variables and the number of pluralities over which these variables range. A language containing plural variables should be interpreted so that the domain is plurally maximal, and the relevant value of 2^κ for determining the number of pluralities on that domain is that yielded by plurally maximal models (for the relevant V_α, namely the least such that a set co-extensional with the first-order domain is an element of V_β for $\beta < \alpha$). Whatever semantic indeterminacy there

[29]In other words, the indeterminacy of concern to us 'kicks in' at $V_{\omega+1}$.

[30]Thank you especially to N for discussion here. For the fleshing out of some consequences of the argument here, and in what follows, see N and N (forthcoming)

might be concerning sets, we have here a plausible candidate for the intended range. For only this interpretation respects the intended meaning of the universal quantifier as all, since in models that are not plurally maximal for V_α there are some pluralities in V_α that do not form a set. This argument will not, of course, convince a sceptical interlocutor, since it is effectively a variation on the "all' means all' move, and assumes what the sceptic will deny, namely that there are some pluralities in V_α that do not form a set. Yet perhaps convincing the sceptic was always a forlorn hope. The line of reasoning allows the supporter of Plural Robustness to shore up her own position and to respond to a certain line of interrogation. To this extent, the dialectic has moved forward. Or so she might think.

Alas, it is not to be. Elementary forcing arguments establish that there is *no* V_α with $\alpha \geqslant \omega + 1$ such that any universe is plurally maximal for V_α. Suppose otherwise. Consider V_α at a maximal universe. α is either a successor or a limit. If the former, consider V_β such that $V_\alpha = \mathcal{P}(V_\beta)$. We now force more subsets of $\mathcal{P}(V_\beta)$, moving to a new universe. Any newly forced subset has as its elements all and only the members of some plurality of elements of V_β, contradicting the supposition that the original universe was plurally maximal for V_α. Alternatively, suppose that α is a limit. Choose some $\beta < \alpha$, and force more subsets of β, moving to another universe. These will be elements of $V_{\beta+1}$ at the new universe, and so elements of V_α, contradicting the supposition that the original universe was plurally maximal for V_α by the same reasoning as in the previous case.

It follows that we cannot secure a categorical plural theory of analysis given set-theoretic pluralism. Nor, of course, can we get a κ-categorical set-theory. Given the history of philosophical interest in plurals, this is a serious loss. Nonetheless it's important to be clear what all of this does, and equally what it doesn't, achieve. It was for this purpose that Boolos first introduced plurals, having originally thought that no higher-order logic could be used to formulate set-theory (Boolos, 1998e)(Boolos, 1998d). The suggestion has been followed up by other authors, with the possibility being mooted of analysing class-talk in plural terms (Boolos, 1998e) (Pollard, 1990) (Uzquiano, 2003). There are limits to the determinacy of plural quantification in a setting of set-theoretic pluralism, even if it is robust relative to set-theoretically interpreted second-order quantification. These limits might be thought unfortunate given the motivations for plural logic advertised at the beginning of the present paper. In due course I will suggest that the loss here is tinged with gain.

3

Recall that **(P)** claims that plural quantification and set-theoretically interpreted second-order quantification are on a par regarding semantic determinacy. Florio and Linnebo argue that the semantic determinacy of plural quantification forces that of quantification over sets. This matters both for the issue of robustness itself and for the question of the determinacy of plural quantifiers, since if Florio and Linnebo are correct then there is an argument by contraposition from set-theoretic indeterminacy (perhaps owing to pluralism) to plural indeterminacy. They consider two cases, one in which the domain is set-sized, and one in which it is not. With respect to the first, they argue as follows:

> Assume that the domain is a set D, and let dd be its elements. (We will indicate this relationship by writing $D = \{dd\}$. In the case of the set-based semantics, we need to single out a special object – the standard interpretation – from a large pool of other objects – the Henkin interpretations. In the case of the plurality-based semantics, we need to single out a special way of ranging over the domain dd – the standard way – from a large pool of other ways of ranging over dd – the Henkin ways. But why should it be any easier – or harder – to single out an object from a pool of objects than to single out a way from an isomorphic pool of ways? Since the two tasks are isomorphic, whatever can be said in one case carries over to the other. (Florio and Linnebo, 2015, 9)

We've already minuted a response that can be made here from the perspective of STP. The issue here is not about distinguishing standard from Henkin interpretations in the case of set-theoretic quantification, but rather about distinguishing between distinct standard interpretations in different universes. The above quoted argument simply doesn't address this line of defence, for which set-theoretic differs from plural quantification with respect to an indeterminacy as to which interpretation is the standard interpretation. Similarly the extension of the argument so as to drop the assumption that the domain is set-sized tacitly assumes that the question about determinacy is exhausted by that about the status of Henkin interpretations:

> Consider. . . the possibility that plural logic is determinate [whereas set-theoretically interpreted second-order logic is not]. If plural logic is determinate. . . this means that no plurality-based Henkin interpretation can be correct. *A fortiori*, no plurality-based Henkin interpretation can be countenanced in which the elements dd of the

> domain form a set *D*. But this is incompatible with the idea that set-based interpretations are legitimate, since the legitimacy of an interpretation would then depend entirely on the way in which the interpretation is described. (Florio and Linnebo, 2015, 9)

Comment is warranted on the suggestion that the defender of determinacy need think that Henkin interpretations are not 'correct', or lack 'legitimacy'. Clearly Henkin interpretations are a useful model-theoretic tool, and the possession of a completeness result is valuable for some purposes. Nobody should deny this. What is at issue is not this, but rather the extent to which the Henkin interpretations should be regarded as being more than algebras for proof systems, and as representing candidate meanings for higher-order quantifiers,[31] or for the natural language and everyday mathematical quantifiers they formalise. It is for this semantic purpose alone that the proponent of determinacy privileges standard interpretations, and thereby executes an argument via categoricity from the determinacy of mathematical language to that of mathematical structure. But as we have seen, the privileging of standard interpretations alone does not secure determinacy if there is any room for indeterminacy about *which* standard interpretation (in which universe) second-order variables range within. So it may be granted that plural and second-order logic are in the same position regarding the semantic priority of standard interpretations, without this forcing the rejection of Robustness.

In addition to this semantic argument Florio and Linnebo supply a metaphysical one (Florio and Linnebo, 2015, 10). If, for a given domain, there is a determinate and maximal property of being a plurality on the domain, it is suggested, then we should conclude that there is a determinate and maximal set of subsets of the domain. The key move here is the proposal that 'once it is granted that a range of objects is determinate, it is hard to see why these objects should not form a set'. Yet this cannot be granted with full generality. For suppose that it is determinate which objects are all and only the non-selfmembered sets.[32] It is not hard to see why *these* objects should not form a set. Prescinding from the paradoxical case: it does not follow simply from the existence of some xx that there is a set whose elements are all and only the x such that $x \prec xx$. A set is something over and above its elements, and that the existence of a set of the xx is automatic given the existence of each of the xx would therefore stand in need of argument even were it not the case that it were simply false

[31] I intend 'higher-order quantifiers' here in an inclusive sense, covering both plural and second-order quantifiers.

[32] Of course, in a context of set-theoretic pluralism, this might not be determinate. But we can simply run the argument that follows with our attention restricted to the non-selfmembered sets in a particular universe.

as a matter of proven mathematical fact. As it is, there is a mathematical science, set theory, which tells us certain purported sets do not exist. It has produced fruitful and applicable mathematics, and in cases of disagreement we therefore have far better reason to believe it concerning set existence than we do any general metaphysical principle about sets. If there is some philosophical doctrine which allows us to move smoothly, perhaps given certain conditions, from the existence of xx to a set of xx then it needs to be motivated by an argument paying due attention to set theory. The prospects for such an argument do not look good (Hewitt, 2015).

The point is, in essence, one made by Boolos some time ago, which was motivational for his own interest in plurals,

> We cannot always pass from a predicate to the extension of the predicate, a set of things satisfying the predicate. We can, however, always pass to the things satisfying the predicate (if there is at least one), and therefore we cannot always pass from the things to a set of them. (Boolos, 1998a, 168)

Of course, this line of argument doesn't carry over to the case of set-sized domains. But here the possibility of set-theoretic pluralism can be invoked again. Let the domain be $\mathbb{N}$. We've already seen that, given STP, there is no 'determinate and maximal set of subsets of the domain', for if there were, that set would be (the genuine) $\mathcal{P}(\mathbb{N})$.

4

How much robustness can the plural logician assume? Some, but perhaps not as much as some philosophers have wanted. Regardless of the truth of STP, we've seen reason to hold that plural formulations of arithmetic are categorical, whilst were STP true set-theoretically interpreted second-order theories would not be. However, if STP is the correct account of set-theoretic reality then there are severe limits to the capacity of plural languages to describe determinate set-theoretic structures. Moreover, plural logic will lack features customarily attributed to standard second-order logic. To see this, let us return to the case of CH.

It is well known that there is a wff in the language of second-order logic that is, on the standard set-theoretic semantics, a logical truth iff CH is true. This is appealed to by Kreisel in support of CH having a determinate truth-value, although this argument has been challenged (Kreisel, 1967) (Weston, 1976). The wff in question makes irreducible use of quantification into polyadic predicate position, which might be thought sufficient to prevent the result carrying over to the case of plural logic[33], but in actual fact polyadic quantification can be implemented in a plural logic by means of plural quantification over tuples, assuming the availability of a pairing function on the domain (which we have in the set-theoretic case)[34].

Following Shapiro's formulation,[35] we are concerned with (Shapiro, 1991, 105):

$$\text{(C)} \qquad \mathrm{C}_{df}: \ \forall X\,(\text{ALEPH-1}(X) \leftrightarrow \text{CONTINUUM}(X))$$

Where the second-order definition of alephs has been presented previously (Shapiro, 1991, 103-5). $CONTINUUM$ is defined:

$$\begin{aligned}\text{(CON)} \quad \exists C \exists R\,[\text{ALEPH-0}(C) \wedge \forall x \forall y (Rxy \rightarrow (Xx \wedge Cy)) \\ \wedge \forall Y (\forall y (Yy \rightarrow Cy) \rightarrow \exists x \forall y (Rxy \leftrightarrow Yy)) \\ \wedge \forall x \forall y (Xx \wedge Xy \wedge \forall z (Rxz \leftrightarrow Ryz)) \rightarrow x = y)]\end{aligned}$$

The issue here is with the dyadic variable R, bound by an existential quantifier. This serves to express a bijection from X onto (in set-theoretic terms) the power set of the denumerably infinite set C. The plural logician is going to translate this in terms of plural quantification over pairs of sets. But this is not enough to secure the determinacy of pluralised (C), since whether there are some pairs which code a bijection between any set satisfying (ALEPH-1) and any set satisfying (CON) is going to depend on whether (CH) is true in the universe over which plural variables range. If it is indeterminate which universe this is (and, goes one line of argument, there is nothing about our set-theoretic *practice* which could secure such determinacy) then the truth-value of pluralised (C) is similarly indeterminate and the Kreisel argument fails. The plural logician is in the same position as the proponent of standard second-order logic.

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[33]The plural logic **PFO** is bi-interpretable with *monadic* second-order logic.

[34]It's worth minuting, however, that the appeal to pairing might not always be in philosophical good shape. Consider the case of a neo-Fregean who wants to make use of the irreducibly polyadic Hume's Principle. Were she to appeal to plural quantification over arithmetic pairs, implemented in terms of prime base exponentiation say, this would be unacceptably question-begging given the epistemological aims of the neo-Fregean project.

[35]Modified for continuity of notation.

This, however, might not be an entirely bad thing. There is, for many philosophers, something embarassing about the standard result that (C) is logical truth iff CH is true. One strand of complaint in the raging debate about the mathematical entanglments of higher-order logic insists that (C) and its ilk ought not to be true *as a matter of logic*.[36] In the context of STP (C) and its plural translation have no determinate truth-value, so at least one line of attack on the basis of entanglement is barred[37]. It might be objected here that the only reason (C) lacks a determinate truth-value is that CH similarly does. So (C) is no less entangled with CH than it is given universism. Whether or not this a good response depends on what exactly the issue with entanglement is supposed to be. Is the worry simply that higher-order logics can *express* mathematical truths? On the face of it this is hardly an objection to such logics: one of the key motivations for the adoption of a higher-order logic is the desire to express notions such as well-ordering, the transitive closure of a relation, and Dedekind-infinitude which cannot be captured using first-order resources alone Shapiro (1991). If entanglement arguments are to have any purchase on the higher-order logician they need to proceed from the common thought that it is definitive of logic that it can be used to reason about any subject matter whatsoever. This being so, purely logical reasoning should be independent of the distinctive claims of the special sciences. In particular, it should not be possible to arrive at a non-logical result by logical means alone. The worry with entanglement is that we might be able to determine the truth of pluralised (C) by work within plural logic. Either way, the entanglement objection looks weak given STP: if we're concerned with whether (C) is decided semantically, then STP should assuage our fear, for in our context there is no determinate fact-of-the-matter to be decided[38]. Alternatively, if the worry is that (C) could be decided by proof *within* the object theory, this is demonstrably not the case: it is provable that (C) is independent of any sound proof theory for second-order logic. This carries across to the plural case.

Is the door to Kreisel style arguments executed in plural logic is definitely closed? Not necessarily, but the prospects for such arguments depend on resolving the debate about STP in favour of the universist. Regardless of issues around plural logic, this debate is one of the most interesting and urgent in the philosophy of set-theory. But given STP, the uses of plural logic in the

[36]Although it is worth noting that (C) is proof-theoretically independent of second-order logic, so there is no sense in which the result implies that CH could be decided by logical means alone. On entanglement see (Jané, 2007).

[37]We'll hereafter omit mention of the plural translation, taking it to be implicit

[38]In actual fact, as Sam Roberts notes, there is a far worse problem for the set-theoretic pluralist who wishes to make use of plural logic, namely that plural comprehension turns out to be false in every universe. Recall the original motiviation for plural logic at the outset of this paper, that it allows principles like Comprehension to be stated as a single axiom! See NN, forthcoming, for reflection on this and the question whether it moves the debate on universism and multiversism forward.

foundations of mathematics are severely constrained. In particular, we lose categoricity for analysis and κ-categoricity for plural ZFC. Meanwhile, absent STP, second-order logic looks no worse off than plural logic for analysis at least and, arguably, for set-theory.[39] Nevertheless, let us allow that multiversism is an epistemic possibility. Then plural logic is relatively robust with respect to this possibility, in that we are assured of the categoricity of plural arithmetic. This may not be all the robustness we hoped for, but it is not nothing.[40]

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[39]Matters are more delicate in the set-theoretic case, since Boolos style arguments against the second-order formulation of set-theory will be apposite here.

[40]Thanks to Neil Barton, John Divers, Øystein Linnebo, Sam Roberts and Robbie Williams for discussions of work discussed here.

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